

A Appendix: Proofs

A.1 Theorem 1 [Base case]

When the tree is unexplored, the greedy solution finds $\arg \max_{\hat{S}} P_{\text{opt}}(\hat{S})$ when exploring at most depth d nodes.

Proof. When exploring at most d nodes, only one solution candidate can be built. We have $P_{\text{best}}(A_k|S_k) \geq P_{\text{best}}(A_j \neq A_k|S_k)$ and $P_{\text{best}}(A_k|S_k) \geq P_{\text{uni}}$ (from As. 1). If a candidate solution contains $P_{\text{best}}(A_k|S_k)$, its expected probability is always higher than any other unexplored candidate solution whose expected probability is uniform (from Eq. (2) and Lemma 1). \square

A.2 Theorem 2 [Inductive step]

Given the set of n^{th} most likely candidate solutions have been explored, the $(n+1)^{\text{th}}$ most likely candidate solution can be inferred by exploring at most d more nodes.

Proof sketch. The n^{th} solution has a probability to yield the optimal solution $P_{\text{opt}}(\hat{S}_n)$ larger than any other unexplored candidate solutions $P_{\text{opt}}(\hat{S})$, $\hat{S} \in \hat{S}_{\text{unexp}}$. Hence, finding the $P_{\text{opt}}(\hat{S}_{n+1})$ is equivalent to finding:

$$\arg \min_{\alpha} (L_{\text{opt}}(\hat{S}_n) - L_{\alpha}) \quad (8)$$

where $L(x) = \log(P(x))$ is the positive log probability and α indicates the dilemmatic state S_{α} to change. Thus, following Eq. (1) the current candidate solution is associated with the log probability:

$$L_{\text{opt}}(\hat{S}_n) = \sum_{k=1}^d L_{\text{best}}(S_k) \quad (9)$$

where $L_{\text{best}}(S_k)$ is the highest log probability. Further, following Eq. (2) + (8), we have:

$$L_{\alpha} = \sum_{k=1}^{\text{depth}(\alpha)-1} L_{\text{best}}(S_k) + L'_{\text{best}}(S_{\alpha}) + \sum_{k=\text{depth}(\alpha)+1}^d L_{\text{uni}} \quad (10)$$

where $L'_{\text{best}}(S_{\alpha})$ is the highest log probability associated with an unexplored child state at S_{α} . Hence, replacing Eq. (10) + (9) in Eq. (8) gives:

$$L_{\text{opt}}(\hat{S}_n) - L_{\alpha} = (L_{\text{best}}(S_{\alpha}) - L'_{\text{best}}(S_{\alpha})) + \sum_{k=\text{depth}(\alpha)+1}^d (L_{\text{best}}(S_k) - L_{\text{uni}}) \quad (11)$$

where the first term corresponds to the difference of log probabilities at the dilemmatic state S_{α} , while the second term is the expected gain in log probability over a uniform log probability distribution in the remaining tree. Once the state S_{α} to be modified has been found, a \hat{S}_{n+1} candidate solution is built by a greedy search in the subtree whose root is S_{α} (see Theorem 1). \square

To simplify the computation, we can approximate $L_{\text{best}}(S_k) - L_{\text{uni}}$ by a positive constant: L_{uni} is constant and $L_{\text{best}}(S_k)$ can be assumed to be constant on average. As a result, Eq. (11) can be approximated by

$$L_{\text{opt}}(\hat{S}_n) - L_{\alpha} \approx L_{\text{best}}(S_{\alpha}) - L'_{\text{best}}(S_{\alpha}) + (d - \text{depth}(\alpha)) \cdot \text{const.} \quad (12)$$

Hence, minimizing Eq. (12) is equivalent to finding:

$$\arg \min_{\alpha} (L_{\text{best}}(S_{\alpha}) - L'_{\text{best}}(S_{\alpha}) - \text{depth}(\alpha) \cdot \text{const.}) \quad (13)$$

where S_α indicates the state in the tree where the next most likely candidate solution should be greedily grown from. The first term favours exploring states where the difference between the best and the second best option is small, while second term gives preference to exploring candidate solutions re-using a large part of the initial solution.

In this document we study the relation between the energy gain and the depth, as well as between the energy gain and the dilemma estimator for decision tree optimization on the ECP2011 and MNIST datasets.

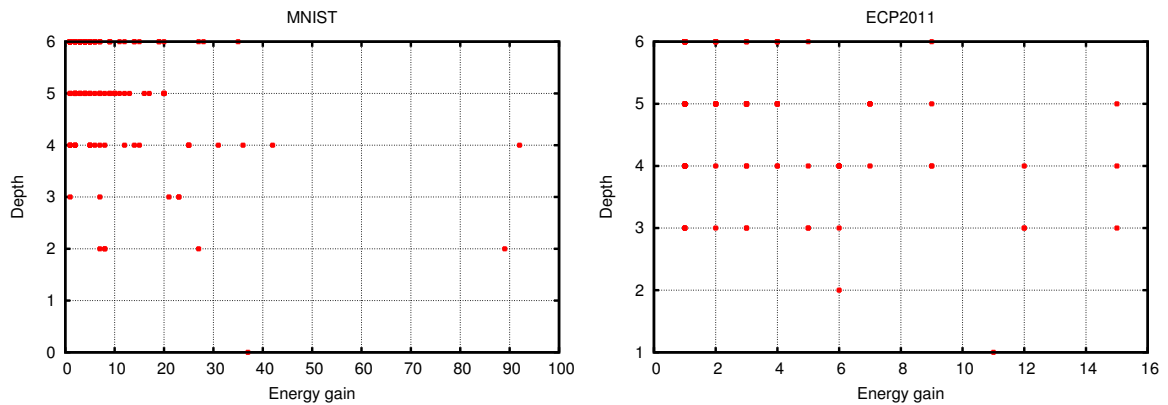


Figure 1: Depth w.r.t. energy gain for the MNIST (left) and ECP2011 (right) datasets

In Fig. 1, we notice that the energy gains obtained for a single iteration show no evidence of correlation with the depth, especially for the ECP2011 dataset. This supports the idea that const. in Eq. (8) can be neglected.

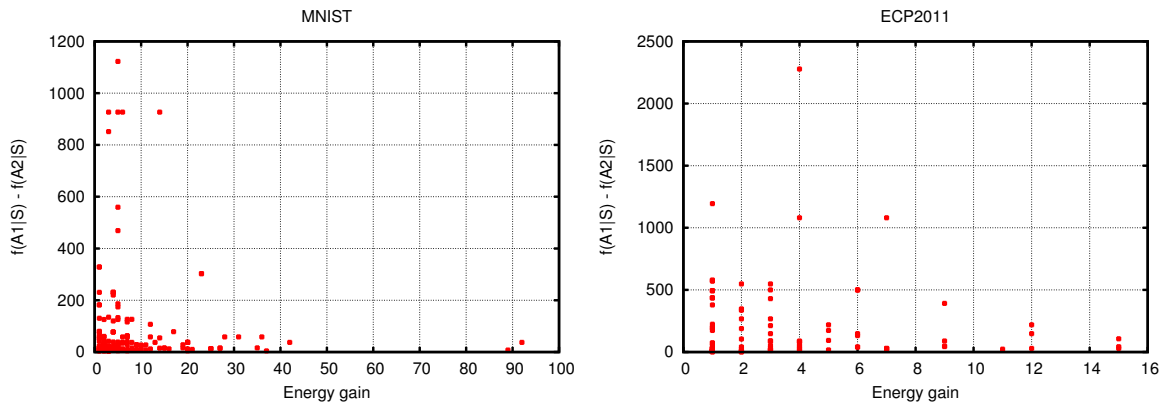


Figure 2: $f(A1|S) - f(A2|S)$ w.r.t. energy gain for the MNIST (left) and ECP2011 (right) datasets

In Fig. 2, we can see that the highest energy gains are obtained for smallest $f(A1|S) - f(A2|S)$ (i.e. larger dilemma estimators). Therefore, we can see how exploring nodes with higher dilemma estimators first helps obtaining higher energy gains first.